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Estimating finite population mean using a new ratio estimator

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ABSTRACT

For the purpose of estimating the finite population mean, a new ratio estimator has been proposed wherein a known coefficient of variation of study variable is used. The proposed ratio estimator is shown to be less biased than the conventional ratio estimator under suitable conditions. The estimator is found to be more efficient than the exponential ratio estimator under condition that holds good in practice very often. Empirical investigations supporting the proposed estimator from the standpoint of percent relative bias or mean squared error have been carried out in respect of several populations, natural and artificial.

Keywords: Finite population, ratio estimator, coefficient of variation, percent relative bias, means squared error

Subject classification code: 2613

1. INTRODUCTION

A finite population consisting of N units such as $U_1, U_2, \ldots\ldots, U_N$ is considered. Let y_i and x_i, i= 1, 2, ..., n, denote the values in respect of study and auxiliary variables, respectively, of the units included in the sample of size n drawn by Simple Random Sampling Without Replacement (SRSWOR) from a finite population. In order to have a survey estimate of the population mean $\bar{Y}$ of the study variable y, assuming the knowledge of the population mean $\bar{X}$ of the auxiliary variable x, Cochran (1940) proposed the classical ratio estimator defined as:

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$$\bar{y}_r = \frac{\bar{y}}{\bar{x}}\bar{X}, \tag{1.1}$$

where the symbols have their usual meanings.

The expressions for Bias and Mean Squared Error of $\bar{y}_r$, to the first degree of approximation, i.e., to $o(n^{-1})$ are given below:

$$B(\bar{y}_r)=\left(\frac{N-n}{Nn}\right)\bar{Y}\left(C_x^2-\rho C_x C_y\right),$$

$$=\left(\frac{N-n}{Nn}\right)\bar{Y}\left(C_{20}-C_{11}\right),$$

$$=\theta_1\bar{Y}\left[C_{20}-C_{11}\right], \tag{1.2}$$

and

$$MSE(\bar{y}_r)=\left(\frac{N-n}{Nn}\right)\bar{Y}^2\left(C_x^2+C_y^2-2\rho C_x C_y\right),$$

$$=\theta_1\bar{Y}^2\left[C_{20}+C_{02}-2C_{11}\right], \tag{1.3}$$

where $\theta_1=\frac{N-n}{Nn}$, $C_{20}=C_x^2$, $C_{02}=C_y^2$ and $C_{11}=\rho C_x C_y$,

$$C_y^2=\frac{S_y^2}{\bar{Y}^2},\ C_x^2=\frac{S_x^2}{\bar{X}^2}\ \text{and}\ C_{rs}=\frac{\mu_{rs}(x,y)}{\bar{X}^r\,\bar{Y}^s},$$

$\mu_{rs}(x,y)$ being the bivariate moment of x and y and other symbols having their usual meanings.

For the purpose of estimating finite population mean $\bar{Y}$, when the variable y is positively correlated with the auxiliary variable x, Bahl and Tuteja (1991) proposed the ratio-type exponential estimator given by

$$\bar{y}_{er}=\bar{y}\exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right]. \tag{1.4}$$

The Bias and Mean Squared Error (MSE) of $\bar{y}_{er}$, to the first degree of approximation, i.e., $o(n^{-1})$ are given by:

$$B\left(\bar{y}_{er}\right)=\theta_1\bar{Y}\left[\frac{3}{8}C_{20}-\frac{1}{2}C_{11}\right] \tag{1.5}$$

and $$MSE\left(\bar{y}_{er}\right)=\theta_1\bar{Y}^2\left[C_{02}+\frac{1}{4}C_{20}-C_{11}\right]. \tag{1.6}$$

Following Searls (1964), Panigrahi and Mishra (2017) have proposed a modified exponential ratio-type estimator of population mean when C_y is known in advance.

The estimator is given by:

$$\bar{y}_{er_1}=\frac{\bar{y}}{1+\theta_1C_y^2}\exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right]. \tag{1.7}$$

The Bias and Mean Squared Error of the estimator, to $o(n^{-1})$ are given below:

$$B\left(\bar{y}_{er_1}\right)=\theta_1\bar{Y}\left[\frac{3}{8}C_{20}-C_{02}-\frac{1}{2}C_{11}\right], \tag{1.8}$$

and $$MSE\left(\bar{y}_{er_1}\right)=\theta_1\bar{Y}^2\left[C_{02}+\frac{1}{4}C_{20}-C_{11}\right]. \tag{1.9}$$

2. PROPOSED ESTIMATOR

Utilizing the ideas mooted by Cochran (1940) and Panigrahi and Mishra (2017), a new ratio-type estimator has been proposed which is compared with the conventional ratio estimator and the conventional exponential ratio estimator, both theoretically and empirically.

The new ratio-type estimator is given by

$$\bar{y}_{r_1}=\frac{\bar{y}}{1+\theta_1C_y^2}\frac{\bar{X}}{\bar{x}}, \tag{2.1}$$

where it is assumed that C_y^2 is known in advance.

The Bias and MSE of the proposed estimator $\bar{y}_{r_1}$, to the first degree of approximation, i.e.,$o(n^{-1})$ are obtained as:

$$B\left(\bar{y}_{r_1}\right)=E\left(\bar{y}_{r_1}-\bar{Y}\right)$$

$$=\theta_1\bar{Y}\left[C_{20}-C_{11}-C_{02}\right] \tag{2.2}$$

and $$\text{MSE}\left(\bar{y}_{r_1}\right)=E\left(\bar{y}_{r_1}-\bar{Y}\right)^2,$$

$$=\theta_1\bar{Y}^2\left[C_{20}-2C_{11}+C_{02}\right]. \tag{2.3}$$

3. COMPARISON OF BIAS

It is observed that the MSEs of the conventional ratio estimator and the proposed ratio estimator are equal. Hence, their biases are compared as follows:

The estimator $\bar{y}_{r_1}$ is less biased than the estimator $\bar{y}_r$ if the condition

$$\left|B(\bar{y}_{r_1})\right| \le \left|B(\bar{y}_r)\right|$$

or, $$\left|\theta_1\bar{Y}\left[C_{20}-C_{11}-C_{02}\right]\right| \le \left|\theta_1\bar{Y}\left[C_{20}-C_{11}\right]\right|$$

or, $$\left|C_{20}-C_{11}-C_{02}\right| \le \left|C_{20}-C_{11}\right| \quad (3.1)$$

If $C_{20} \ge C_{11}$, then condition (3.1) reduces to

$$C_{02} \le 2(C_{20} - C_{11}) \quad (3.2)$$

If $C_{20} < C_{11}$, then (3.1) yields

$$C_{02} \ge 2(C_{20} - C_{11}), \quad (3.3)$$

a condition that always holds.

Precisely speaking, the proposed estimator $\bar{y}_{r_1}$ becomes less biased than the estimator $\bar{y}_r$ if either the conditions $C_{20} \ge C_{11}$ and $C_{02} \le 2(C_{20} - C_{11})$ are satisfied or if $C_{20} < C_{11}$ is met.

4. COMPARISON OF EFFICIENCY

The proposed ratio-type estimator $\bar{y}_{r_1}$ will be more efficient than the conventional exponential ratio estimator $\bar{y}_{er}$ if the condition

$$\text{MSE}(\bar{y}_{r_1}) \le \text{MSE}(\bar{y}_{er})$$

or, $$\theta_1\bar{Y}^2\left[C_{20}-C_{11}+C_{02}\right] \le \theta_1\bar{Y}^2\left[C_{02}+\frac{1}{4}C_{20}-C_{11}\right]$$

or, $$C_{20}-2C_{11}+C_{02} \le C_{02}+\frac{1}{4}C_{20}-C_{11}$$

or, $$\frac{3}{4}C_{20}-C_{11} \le 0$$

or, $$C_{20} \le \frac{4}{3}C_{11} \quad (4.1)$$

is satisfied.

5. EMPIRICAL STUDY

As the MSEs of $\bar{y}_r$ and $\bar{y}_{r_1}$ are equal, percent relative biases of the two estimators are computed for the purpose of comparison. Table 1 presents the description of populations with correlation coefficient (ρ) and the coefficients of variation C_x and C_y .Table 2 gives the absolute biases of different estimators $\bar{y}_r$, $\bar{y}_{r_1}$ and $\bar{y}_{er}$ and Table 3 & Table 4 give the percent relative biases of the estimators $\bar{y}_r$, $\bar{y}_{r_1}$ and $\bar{y}_{er}$ when the sample sizes are 4 and 5, respectively.

For efficiency comparison of proposed ratio estimator $\bar{y}_{r_1}$ and the conventional exponential ratio estimator $\bar{y}_{er}$, Table 5 deals with the description of populations with correlation coefficient (ρ), the coefficients of variation C_x and C_y and Table 6 gives the MSEs of the above two estimators.

Table 1: Description of the Populations

Population No.	Source	X	Y	N	P	C_x	C_y	C_{yx}
1	Draper and Smith (1966 P.366)	Weight Percent of Tricalcium Silicate	Heat Evolved in (Cal/g)	13	0.810	0.323	0.157	0.041
2	Rao (2000 P.218)	Family Size	Television	12	0.590	0.301	0.160	0.028
3	Keshav (2012 P.107)	Day	Mean Ping Time of ISP1	10	0.720	0.551	0.217	0.086
4	Cochran (2015 P.34)	Family Size	Weekly Income	33	0.212	0.403	0.143	0.012
5	Black (2009 P.517)	Job Satisfaction	Advancement Opportunities	19	0.260	0.376	0.438	0.042

6	Black (2009 P.541)	Price of Silver (in dollar / ounce)	Price of Gold (in dollar /ounce)	12	0.720	0.503	0.277	0.100
7	Cochran (2015 P.67)	Number of Males	Number of Females	30	0.048	0.521	0.506	-0.013

Table 2: Absolute Bias of Estimators (Natural & Artificial Populations)

Population No	$\overline{y}_{er}$	$\overline{y}_{r}$	$\overline{y}_{r_1}$
1	0.018	0.063	0.038
2	0.020	0.063	0.037
3	0.071	0.218	0.171
4	0.055	0.150	0.130
5	0.032	0.099	0.093
6	0.045	0.153	0.076
7	0.109	0.284	0.028

Table 3: Percent Relative Bias of Estimators (Natural and Artificial Populations)

Sample size n=4

Population No	$\overline{y}_{er}$	$\overline{y}_{r}$	$\overline{y}_{r_1}$
1	7.488	12.064	7.280
2	5.644	10.430	6.135
3	14.319	19.931	15.635
4	11.703	17.625	15.275
5	3.286	8.791	8.258
6	9.203	17.342	8.630
7	8.761	17.763	1.767

Table 4: Percent Relative Bias of Estimators (Natural and Artificial Populations)

Sample size n= 5

Population No	$\bar{y}_{er}$	$\bar{y}_r$	$\bar{y}_{r_1}$
1	6.318	10.179	6.143
2	4.720	8.721	5.130
3	11.692	16.274	12.766
4	10.259	15.450	13.390
5	2.834	7.583	7.124
6	7.695	14.501	7.216
7	7.689	15.624	1.554

- For all the populations given in Table 3 & 4, the percent relative bias of the estimator $\bar{y}_{r_1}$ is less than that of the estimator $\bar{y}_r$ (for both the sample sizes).
- For the populations 1, 6 and 7 in Table 3 & 4, the percent relative bias of the estimator $\bar{y}_{r_1}$ is less than that of the estimators $\bar{y}_{er}$ (for both the sample sizes).

Table 5: Description of the Populations

Population No	Source	X	Y	N	P	C_x	C_y	C_{yx}
1	Murthy (1967 P.106)	Total number of persons in 1951	Number of workers in industry in 1961	11	0.510	0.593	1.112	0.336
2	Murthy (1967 P.106)	Total area of land in1951 (in square miles)	Number of persons in 1951	11	0.590	0.445	0.594	0.156

Table 6: MSE of Estimators $\bar{y}$, $\bar{y}_{er}$ and $\bar{y}_{r_1}$

(Natural and Artificial Populations)

Population No	$\bar{y}$	$\bar{y}_{er}$	$\bar{y}_{r_1}$
1	1.237	0.989	0.917
2	0.353	0.247	0.239

For both the populations 1, 2 in Table 6, the MSE of the estimator $\bar{y}_{r_1}$ is less than that of both the estimators $\bar{y}_{er}$ and $\bar{y}$.

6. CONCLUSION

As coefficient of variation of a variable remains stable over a period of time, a new ratio estimator, making use of known coefficient of variation of the study variable has been proposed in this paper. Conditions under which the estimator is less biased and more efficient than its competing estimators have been derived. Seven populations, either natural or artificial, have been considered in Table 1, where the proposed estimator $\bar{y}_{r_1}$ has been found to be invariably less biased than $\bar{y}_r$. In respect of a few populations, the proposed estimator $\bar{y}_{r_1}$ comes out to be less biased than its competing estimator $\bar{y}_{er}$. From Table 6, it is also observed that the estimator $\bar{y}_{r_1}$ is more efficient than the estimator $\bar{y}_{er}$ and the simple mean estimator $\bar{y}$ for both the populations.

In view of the above findings, the newly proposed estimator is worth considering, subject to fulfillment of the necessary conditions.

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